Integration by Parts

Consider the function $y = f(x)g(x)$

Find its derivative to develop the formula for integration by parts.

$$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)g'(x) + g(x)f'(x)$$

$$\int \frac{d}{dx}\left(f(x)g(x)\right)dx = \int \left(f(x)g'(x) + g(x)f'(x)\right)dx$$

$$f(x)g(x) = \int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx$$

$$f(x)g(x) - \int g(x)f'(x)\,dx = \int f(x)g'(x)\,dx$$

Write the formula in terms of u and v.

$u = f(x) \quad v = g(x)$

$du = f'(x)dx \quad dv = g'(x)dx$

$$\int u\,dv = uv - \int v\,du$$

The big question is always, "What is u and what is dv?"
The answer lies in paying attention to DETAIL.

v
xponential
rigonometric
lgebraic
nverse trig
ogarithmic

Find $\int x \cos x \, dx$ $= x\sin x - \int \sin x \, dx$

$u = x \qquad dv = \cos x \, dx$

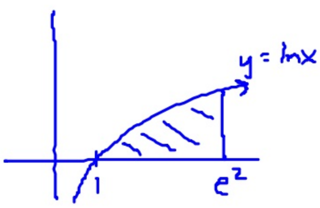
$du = dx \qquad v = \sin x \qquad = x \sin x + \cos x + C$

Evaluate $\int_1^{e^2} \ln x \, dx = x\ln x\Big]_1^{e^2} - \int_1^{e^2} x \cdot \frac{1}{x}\,dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$= x\ln x\Big]_1^{e^2} - \int_1^{e^2} dx$



$= x\ln x - x\Big]_1^{e^2}$

$= (e^2 \ln e^2 - e^2) - (1\ln 1 - 1)$

$= 2e^2 - e^2 + 1$

$= e^2 + 1$

Find $\int x^2 e^x \, dx = x^2 e^x - 2\int x e^x \, dx$

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$= x^2 e^x - 2\left(x e^x - \int e^x dx\right)$

$= x^2 e^x - 2(x e^x - e^x) + C$

$= x^2 e^x - 2x e^x + 2e^x + C$

Find $\int e^t \cos t\, dt$ $= e^t \cos t + \int e^t \sin t\, dt$

$u = \cos t \quad dv = e^t dt$

$du = -\sin t\, dt \quad v = e^t$

$u = \sin t \quad dv = e^t dt$

$du = \cos t\, dt \quad v = e^t$

$$\int e^t \cos t\, dt = e^t \cos t + \left(e^t \sin t - \int e^t \cos t\, dt\right)$$

$$2\int e^t \cos t\, dt = e^t \cos t + e^t \sin t + C_1$$

$$\int e^t \cos t\, dt = \frac{1}{2} e^t \cos t + \frac{1}{2} e^t \sin t + C$$

Calculate $\int_0^1 \tan^{-1} t \, dt = t \tan^{-1} t \Big]_0^1 - \frac{1}{2}\int_0^1 \frac{2t}{1+t^2}\,dt$

$u = \tan^{-1} t \quad dv = dt$

$du = \frac{1}{1+t^2}\,dt \quad v = t$

$u = 1+t^2 \quad t=0 \to u=1$

$du = 2t\,dt \quad t=1 \to u=2$

$$= 1\tan^{-1}1 - 0\tan^{-1}0 - \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln|u|\Big]_1^2$$

$$= \frac{\pi}{4} - \left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln 1\right)$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

alternative correct answers:

- $\frac{\pi - 2\ln 2}{4}$
- $\frac{\pi - \ln 4}{4}$