Continuity

A function is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$

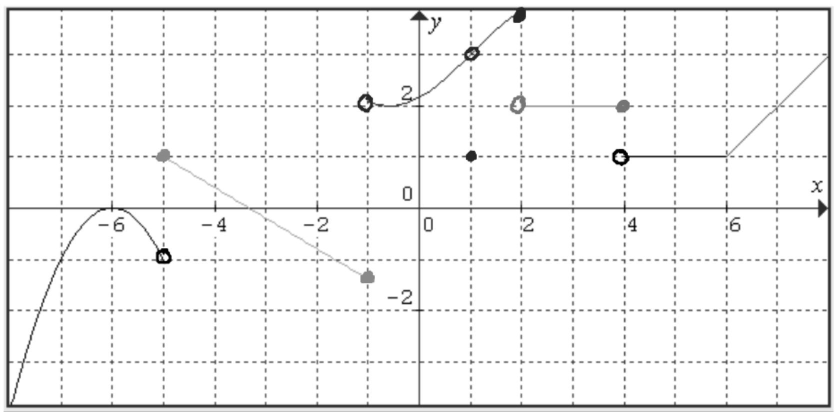
In order for the above statement to hold true,
the following three conditions must be met:

1. $f(a)$ is defined

2. $\lim_{x\to a} f(x)$ exists

3. $\lim_{x\to a} f(x) = f(a)$

Find points of discontinuity, if any, and discuss why you classify it as such.



At $x=-5$, $\lim_{x\to -5} f(x)$ DNE, $\therefore$ f is not continuous at $x=-5$.

(likewise for $x=-1$, 2, and 4)

At $x=1$, $\lim_{x\to 1} f(x) \neq f(1)$, so f is discontinuous at $x=1$.

At $x=-5$, $\lim_{x\to -5^+} f(x) = f(-5)$, so f is continuous at $x=-5$ from the right.

f is continuous on $(-\infty,-5) \cup [-5,-1] \cup (-1,1) \cup (1,2] \cup (2,4] \cup (4,\infty)$

There are a number of different kinds of discontinuities.

The removable discontinuity

The jump discontinuity

The infinite discontinuity (vertical asymptote)

One can have one-sided continuity, where either

$$\lim_{x\to a^-} f(x) = f(a) \quad \text{or} \quad \lim_{x\to a^+} f(x) = f(a)$$

for continuity from the left or from the right, respectively.

Definition: A function f is **continuous on an interval** if it is continuous at every number in the interval.

If f and g are continuous at a, and if c is a constant, then the following are also continuous at a:

1. $f+g$ 2. $f-g$ 3. cf

4. fg 5. $\frac{f}{g}$ if $g(a)\neq 0$

Any polynomial function is continuous everywhere, that is, on $(-\infty,\infty)$

Any rational function is continuous whenever it is defined, that is, on its natural domain.

In fact, the following functions are continuous on their domains:

polynomials	rational functions
root functions	trigonometric functions
exponential functions	logarithmic functions

Composition of functions and continuity

If f is continuous at b and $\lim\limits_{x \to a} g(x) = b$,

then $\lim\limits_{x \to a} f(g(x)) = f(b)$

If g is continuous at a and f is continuous at $g(a)$ then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

The Intermediate Value Theorem (IVT)

If f is continuous on the closed interval $[a,b]$, then if N is any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$, then there exists a number c in the open interval (a,b) such that $f(c) = N$.

Show that there must be a value c on the interval $1 < x < 3$ such that $h(x) = -5$ if $h(x) = 0.1x^4 - 1.3x^3 + 2x^2 + 1$

$h(1) = 1.8$

$h(3) = -8$

Since h is a polynomial function, it is continuous on $[1,3]$.

Since $h(3) < -5 < h(1)$, there exists a value c on $(1,3)$ such that $h(c) = -5$.