The Limit of a Function

When we found the slope of the tangent line by looking at secant line slopes as point Q moved closer and closer to point P, we can describe the value that was approached as the *limit*.

In general, the limit of a function $f(x)$, as x approaches a constant a, equals the limit L. Symbolically, we write

$$\lim_{x \to a} f(x) = L, \text{ where } L \in \{\text{reals}\}$$

We can find a limit numerically using technology, just as we did for estimating the slope of the tangent line. Estimate the following limit.

$$\lim_{x \to 3} \frac{x-3}{x^2-9} = \frac{1}{6}$$

Find the following limits using technology.

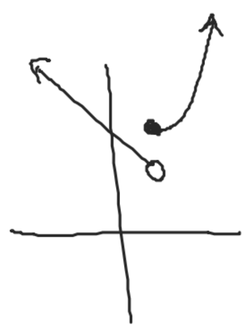
1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$

2. $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$ Does Not Exist DNE

Notice that the second limit does not exist. This happens because the values continue to oscillate between -1 and 1, no matter how close we get to 0, and therefore the values don't approach any particular number as x approaches 0.

3. $\lim_{x\to 2} f(x)$ for $f(x) = \begin{cases} x^2+3, & x \geq 2 \\ 12-3x, & x < 2 \end{cases}$

DNE

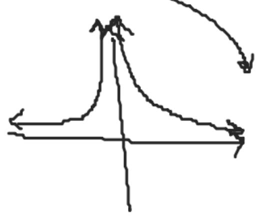


For a limit to exist, the limits from either side must exist and be equal, i.e.,

$$\lim_{x\to a} f(x)=L \text{ iff } \lim_{x\to a^-} f(x)=L \text{ and } \lim_{x\to a^+} f(x)=L$$

4. $\displaystyle\lim_{x\to 0}\frac{2}{x}$ DNE

5. $\displaystyle\lim_{x\to 0}\frac{2}{x^2}$ DNE or ∞



If one or both sides of a limit approach infinity or negative infinity as x approaches a, the graph will have a vertical asymptote at $x = a$.