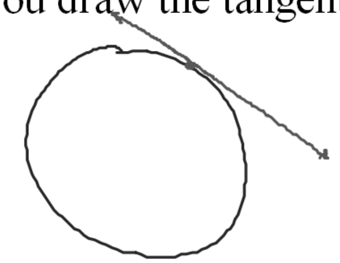
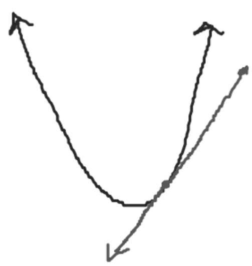
The Tangent and Velocity Problems

What is a tangent line?

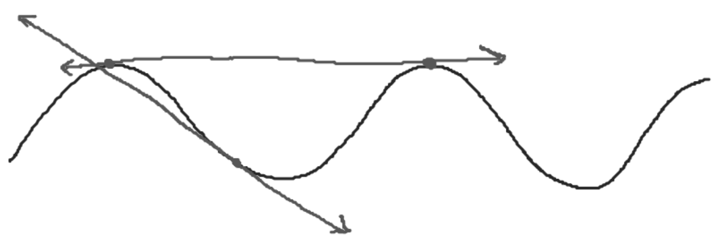
a line that touches a figure at only one point (?)

Can you draw the tangent to a circle? A parabola?



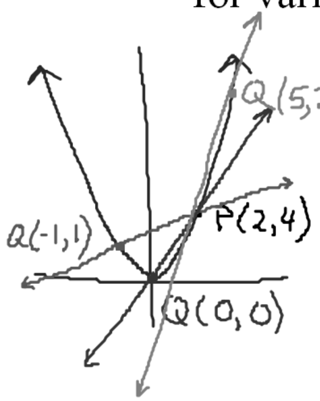


How about for a sinusoid?



Find the slope of the tangent line to the parabola $y = x^2$ at the point $P(2,4)$.

Start with a secant line and find its slope m_{PQ} for various points Q.



$m_{PQ} = \frac{4-1}{2-(-1)} = 1$

$m_{PQ} = \frac{4-0}{2-0} = 2$

$m_{PQ} = \frac{25-4}{5-2} = 7$

use a generic point Q (x, x^2)

$m_{sec} = \frac{4-x^2}{2-x}$

As point Q approaches (nears) point P, what happens to the slope value?

as $Q \to P$, $m_{sec} \to m_{tan}$

$\therefore m_{tan} = 4$

Eq. of tangent line:

$y - 4 = 4(x-2)$

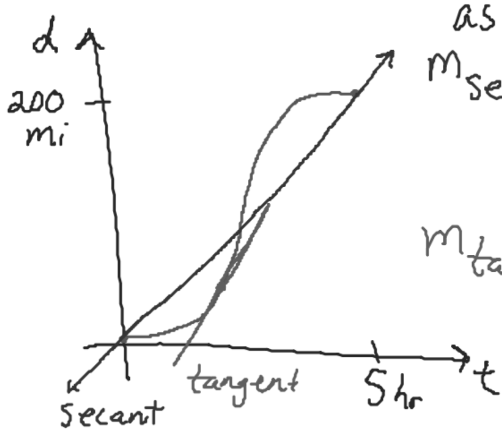
Find the average velocity of a car which moves 200 miles in 5 hours.

$$v_{avg} = \text{(distance traveled)/(time elapsed)}$$

$$= \frac{200 \text{ mi}}{5 \text{ hr}} = 40 \text{ mph}$$

Look at a sketch of a graph of time versus distance. If we want to know the instantaneous velocity at a certain time, how could we do this? Use the same procedure we just learned.

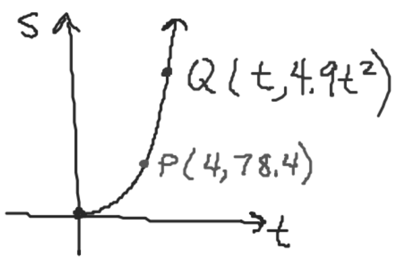
as $Q \to P$, $m_{sec} \to m_{tan}$ (velocity)



$m_{sec} \sim$ avg. velocity
(average rate of change)

$m_{tan} \sim$ velocity
(instantaneous rate of change)

Suppose a rock is dropped from the top of a building and that the distance it has traveled at time t can be expressed as $s(t) = 4.9t^2$. Find the velocity of the rock after 4 seconds.



$$m_{sec} = \frac{4.9t^2 - 78.4}{t - 4}$$

as $Q \to P$,

$m_{sec} \to m_{tan}$ (velocity)

$\therefore m_{tan} = \text{velocity} = 39.2\ \frac{m}{s}$