Equations

When solving an equation, we are finding the set of real numbers which makes the equation true. We will write a solution set for each solved equation.

Solve the following.

1. $2x - 7 = 12$

$$2x = 19$$
$$x = \frac{19}{2}$$
$$S = \left\{ \frac{19}{2} \right\}$$

2. $x^2 = 25$

$$x^2 - 25 = 0$$
$$(x+5)(x-5) = 0$$
$$x+5=0 \quad \text{or} \quad x-5=0$$
$$x=-5 \quad \text{or} \quad x=5$$
$$S=\{-5, 5\}$$

3. $5x - 7 = 2x + 4$

$5x = 2x + 11$

$3x = 11$

$x = \frac{11}{3}$

$S = \{ \frac{11}{3} \}$

4. $(5x + 3)(2x - 1) = 0$

$5x + 3 = 0$ or $2x - 1 = 0$

$5x = -3$ or $2x = 1$

$x = -\frac{3}{5}$ or $x = \frac{1}{2}$

$S = \{ -\frac{3}{5}, \frac{1}{2} \}$

5. $|x + 3| = 5$

$x + 3 = -5$ or $x + 3 = 5$

$x = -8$ or $x = 2$

$S = \{ -8, 2 \}$

6. $x = 3$

$S = \{ 3 \}$

What if we were to multiply both sides of the equation in #6 by x?

$x^2 = 3x$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x - 3 = 0$

$x = 3$

$S = \{ 0, 3 \}$

The result is that an *extraneous* solution is created (one <u>not</u> valid in the original equation), and it must be discarded.

7. $x^2 = 8x$

$x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0$ or $x - 8 = 0$

$x = 8$

$S = \{0, 8\}$

What if we were to divide both sides of the equation in #7 by x?

$x = 8$ $S = \{8\}$

The result is that a valid solution for the original equation is lost. This is a very bad error.

Sometimes the domain of a problem is restricted. When this is done, all algebraically valid solutions must pass through a filter (the domain restriction) before the solution set is written.

For example, solve the following using a domain $D = \{\text{positive numbers}\}$

$$3x + 6 = -5$$

$$3x = {}^{-}11$$

$$x = \frac{-11}{3}$$

$$S = \emptyset$$